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$$3\mu^2a^2=[\mu a+Q\sin(\phi-\theta)]^2+[\mu b+Q\sin(\theta+\tfrac{1}{2}\beta-\phi)]^2$$
$$+2[\mu a+Q\sin(\phi-\theta)][\mu b+Q\sin(\theta+\tfrac{1}{2}\beta-\phi)]\cos\tfrac{1}{2}\beta\ldots(4).$$

Eliminating ϕ between (3) and (4) gives an equation to determine θ.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

152. Proposed by H. S. VANDIVER, Bala, Pa.

When p is a prime of the form $5n\pm1$ then there is a positive integer a such that $a^2\equiv5(\text{mod } p)$. Show that $\left(\dfrac{a\pm1}{p}\right)=\pm\left(\dfrac{-2a}{p}\right)$, according as p is of the form $5n+1$ or $5n-1$.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A general expression involving all cases is, apparently, not easily deduced. The following cases hold.

(1) Let $n=2$. Then $p=5n+1=11$. $a=4$, $a=p-4=7$, $a=2p-7=15$, $a=3p-15=18$, etc.

(2) Let $n=4$. Then $p=5n-1=19$, $a=9$, $a=p-9=10$, $a=2p-10=28$, $a=3p-28=29$, etc.

(3) Let $n=6$. Then $p=5n+1=31$, $a=6$, $a=p-6=25$, $a=2p-25=37$, $a=3p-37=56$, etc.

(4) Let $n=8$. Then $p=5n+1=41$, $a=13$, $a=p-13=28$, $a=2p-28=54$, $a=3p-54=69$, etc.

(5) Let $n=12$. Then (b) $p=5n+1=61$; (c) $p=5n-1=59$.

(b) $a=26$, $a=p-26=35$, $a=2p-35=87$, $a=3p-87=96$, etc.

(c) $a=8$, $a=p-8=51$, $a=2p-51=67$, $a=3p-67=110$, etc.

For every value of n that makes $5n\pm1$ a prime, we can find values for a satisfying the condition. It is also easy to see that a can have an infinitude of values for each case.

In (1), (3), (4), (5)(b), $(a+1)^{\frac{1}{2}(p-1)}=(-2a)^{\frac{1}{2}(p-1)}\equiv-1(\text{mod. } p)$.

In (2), (5)(c), $(a-1)^{\frac{1}{2}(p-1)}=-(-2a)^{\frac{1}{2}(p-1)}\equiv1(\text{mod. } p)$.

$\left(\dfrac{a\pm1}{p}\right)=\pm\left(\dfrac{-2a}{p}\right)$, in the cases examined, according as $p=5n\pm1$.

A general solution is desired. ED. F.

152. Proposed by H. S. VANDIVER, Bala, Pa.

Prove geometrically:

$$\sum_{n=1}^{\frac{1}{2}(p-1)}\left[\frac{n^2}{p}\right]=\frac{p-3}{4}\cdot\frac{p-1}{2}-\sum_{n=1}^{\frac{1}{4}(p-4}\left[\sqrt{np}\,\right],$$ where $p\equiv3(\text{mod. } 4)$ and $\left[\dfrac{k}{p}\right]$ represents the greatest integer in k/p.